Multiple prediction using the homeomorphic-imaging technique

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Abstract
A new method for predicting different kinds of multiples and peg-leg reflections in unstacked seismic data is discussed. The basis for this method is the fact that kinematic properties of multiples can be represented as a combination of kinematic properties of primary reflections. The prediction is made using a two-step process. In the first step, the values for the angle of emergence and radius of curvature of the wavefront for primary reflections from ‘multiple-generating’ interfaces are obtained. These parameters are estimated directly from unstacked data for every source point using the homeomorphic-imaging technique. The second step consists of prediction of multiples from primary reflections that satisfy a so-called ‘multiple condition’. This condition is the equality of the absolute values of the angles of emergence calculated from the first step. This method is effective even in complex media and information on the subsurface geology is not required. The parameters are estimated directly from the unstacked data and do not require any computational efforts such as in wavefield extrapolation of data.

Introduction
Multiple-suppression techniques make use of the different characteristics that distinguish multiple from primary reflections. The characteristics most frequently used are the moveout difference between multiple and primary and the periodic nature of multiples due to the fact that multiples are repetitions of some primary reflection. The first characteristic may be exploited to attenuate the multiples by stacking the data (Schneider, Prince and Giles 1965) or moveout filters (Hardy, Warner and Hobbs 1989; Yilmaz 1989). Short-period multiples are approximately periodic and may be discriminated using predictive and adaptive deconvolution methods (Backus 1959; Peacock and Treitel 1969; Griffiths, Smolka and Trembly 1977). Another common approach is suppression based upon wave-equation methods (Berryhill and Kim 1986; Wiggins 1988; Fokkema and Van den Berg 1990; Verschuur and Berkhout 1992). The wave-equation methods produce a model of water-bottom peg-leg multiples by extrapolating the recorded data through the water layer and subtracting it from the

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data. The traveltime through the water layer is required to produce an accurate time prediction and an estimate of the water-bottom reflectivity function is required to calculate multiple amplitudes.

Not all the proposed approaches lead to a ‘panacea solution’ for multiple attenuation. Even the most powerful and comprehensive methods cannot remove all types of multiples. We introduce a new target-orientated approach to predict kinematic properties of any kind of multiple for a given primary reflection from ‘multiple-generating’ interfaces. Unlike wave-equation extrapolation methods, it does not require any *a priori* knowledge of the subsurface geology or waveform information and does not require significant computational effort to perform the wavefield extrapolation of prestack seismic data. The concept of this approach is explained in the following sections and the method is demonstrated using both synthetic and field examples.

**Basics (Geometrical considerations)**

We describe here the main basic concepts of the method. Figure 1 depicts three reflectors $n$, $n-1$ and $n-2$ beneath an arbitrary overburden. A raypath $A_sC_1C_2C_3A_kC_4A_r$ of the interlayer peg-leg multiple is shown connecting one source $A_s$ with one receiver $A_r$. We can see that the arrival time of this kind of multiple can be represented as a combination of four primary reflections; from reflector $n$, $n-1$ and two reflections from reflector $n-2$. From the above, the arrival time for the interlayer peg-leg multiple

![Ray scheme for an interlayer peg-leg multiple event](https://example.com/ray_scheme.png)

**Figure 1.** Ray scheme for an interlayer peg-leg multiple event $A_sC_1C_2C_3A_kC_4A_r$. The arrival time of this multiple is a simple combination of arrival times of the primary reflections, namely $A_sC_1A_m$, $A_mC_2A_l$, $A_lC_3A_k$ and $A_kC_4A_r$. 

can be given by
\[ T_{n,(n-1)(n-2)} = T_{n,m}^{n} + T_{n-1}^{n} - T_{n-2}^{n} + T_{n-2}^{n-2}. \]

where \( T_{n,m}^{n} \) is the traveltime of waves reflected from interface \( n \) with the source at \( A_s \) and receiver \( A_m \); \( T_{n-1}^{n} \) is the traveltime of waves reflected from interface \( n-1 \) with the source at \( A_k \) and receiver \( A_m \); \( T_{n-2}^{n} \) is the traveltime of waves reflected from interface \( n-2 \) with the source at \( A_l \) and receiver \( A_m \); and \( T_{n-2}^{n-2} \) is the traveltime of waves reflected from interface \( n-2 \) with the source at \( A_k \) and receiver at \( A_l \). When the order of multiple increases by one, it gives rise to an additional reflection. In general it can be shown that any multiples of any order can be represented as a combination of primary reflections.

**Multiple condition**

We can now formulate conditions for multiple prediction for interlayer peg-leg multiples. From Fig. 1, it is evident that the angle of emergence of the wavefront of the primary reflection \( A_s C_1 A_m \) from reflector \( n \) is equal to the angle of emergence of the primary reflection \( A_l C_2 A_m \) from reflector \( n-2 \), the angle of emergence of primary reflection \( A_k C_3 A_l \) from reflector \( n-1 \) is equal to the angle of emergence of primary reflector \( A_m C_2 A_l \) from reflector \( n-2 \), and from Snell’s law it stems that the angle of emergence of the wavefront of primary reflection \( A_l C_3 A_k \) from reflector \( n-1 \) is equal to the angle of emergence of primary reflection \( A_s C_3 A_k \) from reflector \( n-2 \). These conditions can be expressed in the following equations:

\[
\beta_{n,m}^{n} = \beta_{n-2}^{n,m},
\beta_{n-1}^{n-1} = \beta_{n-2}^{n-1},
\beta_{n-1}^{n-2} = \beta_{n-2}^{n-1}.
\]

where \( \beta \) is the angle of emergence of the wavefront, the upper index denotes the number of the reflector, the first bottom index denotes the location of the shot and the second the location of the receiver.

We shall refer to expressions (2) as a ‘multiple condition’.

**Prediction procedure**

For a given type of multiple, the procedure of multiple predictions consists of the following steps:

1. The angle of emergence and radius of curvature for every offset (receiver position) of the wavefront of the primary reflection from multiple-generating interfaces are obtained for each common-shot gather.
2. For a given source-receiver location all possible primary reflections that satisfy the multiple condition are selected (thereby defining the points \( A_s, A_m \) and \( A_k \) in Fig. 1).
3 Calculation of the arrival times for a given source-receiver location from primary reflections.

The crucial step in our prediction method is the derivation of the angle of emergence of the wavefront of a given primary reflection for each shot at every receiver location. Several algorithms have been suggested for this purpose (Shultz and Claerbout 1978; Biondi 1992). They all calculate the angle of emergence (or \( p \)-parameter) using zero-offset arrival times and the stacking velocity. Such a calculation usually requires numerical differentiation procedures which are very sensitive to errors in estimated parameters. We estimate these parameters directly from unstacked data using the common-shot homeomorphic-imaging (HI) method. It requires no more than that the near-surface values of the velocity field be known.

The concept of this method is explained briefly in the following section.

\[ A_0 \]
\[ V_0 \]
\[ D_k \]
\[ R_0 \]
\[ C_k \]
\[ E_k \]
\[ E_0 \]
\[ S \]
Common-shot homeomorphic-imaging method

The estimated angle of emergence and radius of curvature of the wavefront of primary reflections are based on measuring the coherence of a common shot (common-receiver gather) using the common-shotpoint (CSP) HI method (Gelchinsky 1989; Keydar, Gelchinsky and Berkovitch 1996). The basis of the CSP HI method is a new local moveout time correction. As indicated above, the CSP HI method does not require a knowledge of the overburden. All that is needed, even for a laterally inhomogeneous true subsurface model, is the near-surface velocity \( V_0 \), assumed to be known already and constant in the vicinity of each source-point location.

Figure 2 shows the basic ray diagram of the CSP method.

Let us assume that a normal incident ray is emitted at source point \( A_0 \), reflected from an interface \( S \) at point \( C_0 \) and emerges at \( A_0 \) (Fig. 2) at time \( t_0 \). Let us further assume that another ray emitted at source \( A_0 \) is reflected from the interface \( S \) and emerges at

Figure 3. A flow chart of the proposed prediction procedure.
point \( A_0 \) at time \( t_k \). We denote the angle of emergence of the normal ray at \( A_0 \) by \( \beta_0 \) and the radius of the CS wavefront at \( A_0 \) by \( R_0 \).

One of the key ideas of the CSP HI method is to substitute the laterally inhomogeneous true subsurface model by the effective model. This consists, in respect of the CS gather at \( A_0 \), of a homogeneous half-space. We then assume that the true CS wavefront, \( \Sigma \), is approximated in the vicinity of \( A_0 \) by an effective wavefront with the same radius of curvature, \( R_0 \), and the same angle of emergence, \( \beta_0 \). The effective wavefront relates to propagation in the effective medium with its evolute (locus of the centres of curvature of the wavefront) at \( E \) surrounding point \( E_0 \). The element \( E \) is called a homeomorphic image of the element of reflector \( C_0 C_k \). The total set of images on the continuum of central points \( A_0 \) constructs an effective reflector that corresponds to reflector \( S \). The time correction \( \Delta t_k \) for the arbitrary \( k \)th trace corresponding to a source \( A_0 \) and receiver \( A_k \) is described by the relationship,

\[
\Delta t_k = \tau(A_0 C_k A_k) - \tau(A_0 C_0 A_0) = D_k A_k / V_0,
\]

where \( V_0 \) is the velocity at the reference level, constant in the vicinity of \( A_0 \); \( A_0 C_k A_k \) is the travelpath from source \( A_0 \) to receiver \( A_k \); \( D_k A_k \) is the difference in travelpath between zero-offset trace \( A_0 C_0 \) and the \( k \)th trace \( A_0 C_k A_k \). By choosing a different order of approximation for the front caustic \( E_0 \), various parametrized relationships for the segment \( D_k A_k \) can be obtained. The order of approximation depends on the distance between the central point location and the furthest receiver. If \( A_0 A_k \) is large, the approximation fails. For a wide class of models and moderate offsets, the wavefront element can be approximated as the arc of a circle. This means that the caustic of the front shrinks to a point and, in such a case, the angle of emergence \( \beta_0 \) of the normal emergence ray \( A_0 C_0 \) and the radius of wavefront curvature \( R_0 \) at point \( A_0 \) can be used

\[ \begin{align*}
A_0 & \quad \Delta t_k \quad A_k \\
\beta_0 & \quad \beta_k \quad R_0
\end{align*} \]

**Figure 4.** Using the estimated angles of emergence \( \beta_0 \) and radii of curvature \( R_0 \) of the wavefront for normal incidence, we use a simple relationship (5) to obtain the angle of emergence \( \beta_k \) for different receiver positions. \( \Delta t_k \) is the offset distance between the zero-offset located at \( A_0 \) and a receiver located at \( A_k \).
to compute the local NMO correction. Assuming a circular wavefront and a locally flat recording plane the following simple relationship is obtained:

$$\Delta t_x = \sqrt{R_0^2 + 2R_0 \Delta x \sin \beta_0 + \Delta x^2} - R_0/V_0,$$

where $\Delta x$ is the distance between source $A_0$ and receiver $A_k$. The unknown parameters, the angle of emergence $\beta_0$ and radius of curvature $R_0$, can be estimated using the wave-correlation procedure (Tamer and Koehler 1969; Gelchinsky, Landa and Shtivelman 1985). The search is performed around the zero-offset time of a specified primary reflection (interpreted from a stacked section). The near-surface velocity $V_0$ should either be known or can be one of the search parameters. Semblance maximization is carried out using an optimization algorithm. Time correction is applied for each central trace located at the source location.

**Implementation**

According to the scheme described above, the method for prediction of multiples is implemented as follows (Fig. 3):

Once the angles of emergence and the radii for normal incidence of the wavefront

![Figure 5. A simple five-layer shallow water model (after Zhou and Greenhalgh 1996).](image)
have been estimated for all shot positions, we can obtain the angle of emergence for different receiver positions (offset) from the shot using the following expression (Fig. 4):

\[
\sin \beta = \frac{\Delta x + R_0 \sin \beta_0}{\sqrt{R_0^2 + 2R_0 \sin \beta_0 \Delta x + \Delta x^2}}.
\]  

(5)

Figure 6. The synthetic CSP gather created for the model in Fig. 5 using acoustic modelling. The primary reflections from B (the sea-bottom), R₁, R₂ and R₃ are labelled on the section. The water-layer reverberations corresponding to the reflector B are BM₁, BM₂, BM₃, the first-, second- and third-order surface multiples, respectively. The symbols R₁M₁, R₁M₂ on the seismic section stand for the first- and second-order peg-leg multiples from reflector R₁. The peg-leg multiples from reflector R₂ are R₂M₁ and R₂M₂. Only the first-order multiple R₂M₁ from reflector R₂ can be observed. Primaries are indicated by solid lines and multiples by dashed lines. As the order increases, so the dashes become smaller. The primary reflections from B and multiples corresponding to this reflector are shown in red. Primary reflections from R₁ and its multiples are shown in green; primary reflections from R₂ and its multiples are shown in orange and the primary reflections from R₃ and corresponding multiples are shown in dark green.
Thus, for a given source receiver, using these angles of emergence, we can define the locations of points which satisfy the multiple condition (points $A_k$, $A_l$, and $A_m$ in Fig. 1). This can be achieved by scanning all possible locations. The traveltime of the multiple reflection for the given source receiver can now be predicted using (1) for interlayer peg-leg multiples. The arrival time of the primary reflection for a source located at $A_0$ from reflector $n$ and for a receiver located at $A_k$ is calculated using (4) and the zero-offset arrival time for this source. From the above, the following expression for an interlayer peg-leg multiple is obtained:

\[
T_{sr}^{n(n-1)(n-2)} = T_{sm}^{n} + T_{kl}^{n-1} - T_{lm}^{n-2} + T_{kr}^{n-2}
\]

\[
= T_{sm}^{n} + \Delta \tau_{sm}^{n} + T_{kl}^{n-1} + \Delta \tau_{kl}^{n-1} - T_{lm}^{n-2} - \Delta \tau_{lm}^{n-2} + T_{kr}^{n-2} + \Delta \tau_{kr}^{n-2},
\]

where $T_{sm}^{n}$, $T_{kl}^{n-1}$, $T_{kr}^{n-2}$ and $T_{lm}^{n-2}$ are the zero times for points $A_s$, $A_k$, $A_l$ for reflectors $n$, $n-1$ and $n-2$; $\Delta \tau_{sm}^{n}$ is the time correction for shot $A_s$ and receiver $A_m$ for reflector $n$; $\Delta \tau_{kl}^{n-1}$ is the time correction for shot $A_k$ and receiver $A_l$ for reflector $n-1$; $\Delta \tau_{lm}^{n-2}$ is the time correction for shot $A_l$ and receiver $A_m$ for reflector $n-2$; $\Delta \tau_{kr}^{n-2}$ is the time correction for shot $A_k$ and receiver $A_r$ for reflector $n-2$. (The first bottom index denotes the location of the shot, the second the location of the receiver and the upper index is the number of the reflector.)

![Figure 7. Angle of emergence of wavefronts for the sea-bottom B, and the second and third reflectors, R_2 and R_3, as a function of offsets obtained using (6).](image)

Synthetic examples

A five-layer shallow water model described by Zhou and Greenhalgh (1996) was used to test our method of multiple prediction (Fig. 5). A CSP record was calculated using acoustic modelling (Schmidt and Tango 1986) (Fig. 6). The distance between receivers was 10 m and the sampling rate was 4 ms. The identification of waves is given in the caption. The CSP HI procedure for analysing the normal rays was applied for the sea-bottom and three reflectors. The angles of emergence of the wavefronts at zero-offset were equal to zero degrees and the estimated values of the radii of curvature of the wavefronts from the bottom, second and third reflectors had an error of less than 4%. Figure 7 is the extrapolated angle of emergence of wavefronts from the bottom, second and third reflectors for different offsets. We can see that the values extrapolated essentially coincide with the true values.

A surprising result was obtained for the first reflector. The error in the estimated radius of curvature was about 25%. Testing the CSP record (Fig. 6), we noted that the source of the error is contamination by the first-order multiple (BM_1), which has the same zero-offset time as the primary from the first reflector. An acceptable error of less

Figure 8. Angle of emergence of the wavefront for the first reflector obtained using the radius of curvature of the wavefront at zero-offset before sea-bottom suppression (long-dash line) and after sea-bottom suppression (solid line). The actual angle of emergence is indicated by the short-dash line.

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than 3% on the radius of curvature for the first reflector was only obtained after applying a \( \tau - p \) multiple suppression filter, using predicted traveltimes for sea-bottom multiple (BM1), to remove contamination of the primary event (R1) by the sea-bottom multiple (BM1). In general, where ambiguity exists between a primary and a multiple, which is some form of conventional multiple, suppression may be applied prior to the application of the proposed prediction procedure. Although there is interference between reflector R3 and the sea-bottom multiple BM2, the error in the estimated radius is good (less than 3%) owing to the fact that the zero-offset times of both events are different.

The extrapolated angles of emergence for the first reflector before and after bottom-multiple suppression are shown in Fig. 8. Using the angles of emergence of primary reflections and satisfying the multiple condition, arrival times of sea-bottom multiples, multiples from the first reflector, multiples from the second reflector and multiples from the third reflector were obtained (Fig. 6, dashed coloured lines). Now, from the predicted traveltimes, the multiple model can be constructed followed by multiple subtraction (Zhou and Greenhalgh 1996; Landa, Belfer and Keydar, submitted to Geophysics).

In order to demonstrate the generality of the proposed prediction procedure, a more complex model including a dipping sea-bottom with a dipping basalt horizon was used.

![Figure 9. Model with dipping sea-bottom and dipping basalt horizon.](image)

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(Fig. 9). A total of 100 common-shot records were calculated using full-wave equation modelling. Each common-shot gather consists of 48 receivers. The distance between receivers was 40 m, the same as the distance between sources. The minimum offset was 40 m. The sampling rate was 2 ms.

A typical CSP gather is shown in Fig. 10. The reflected wave from the sea-bottom appears at a zero time of about 0.4 s, and the reflection from the basalt layer appears at 0.7 s. The aim of this example is to predict the first- and second-order sea-bottom multiples and the first- and second-order peg-leg multiples from the basalt horizon. In order to perform the prediction, the CSP HI method was applied along zero-offset times of the first and second reflectors to estimate the angles of emergence and radii of wavefronts from these reflectors. The results of estimated radii of wavefront curvature from the first and second layers are shown in Fig. 11. The rms errors in angle
estimation for both layers were less than 1%. The rms errors in radii estimation were about 3%.

In the next step the angles of emergence for normal rays were extrapolated at every offset using (5). Finally the traveltimes of multiples were calculated from the primary reflections that satisfy the multiple conditions. The predicted arrival times of four multiples (two sea-bottom and two peg-leg) from the basalt horizon are shown in Fig. 10 by solid lines. A good fit between the predicted times and those calculated by modelling can be observed.

Real data example

The multiple prediction procedure was applied to a marine seismic line. The data were recorded using a 120-channel hydrophone streamer with a 25 m group interval and minimum offset of 139 m. The data were recorded at a 4 ms sampling rate. The first water-layer reverberation is visible at about 0.8 s on a conventional data stack (Fig. 12). The other water peg-leg multiple of the first order from reflector M (about 1.8 s) is present at about 2.2 ms. For this data set, we apply our multiple prediction procedure to predict multiple arrivals of the first water-layer reverberation and first-order peg-leg

![Figure 11. The estimated and actual radii of curvature of the wavefront from the first layer (dashed and solid lines) and the second layer (small and large dots).](image)

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multiples from reflector M (1.8 s). For this purpose the angles of emergence and radii of curvature of the wavefront from the sea-bottom and reflector M were obtained using our procedure. the radii and angles of emergence for the sea-bottom are shown in Figs 13(a,b) and the radii and angles of emergence for reflector M are shown in Figs 14(a,b). The predicted zero-offset times of the first sea-bottom multiple and a strong multiple produced by reflector M are shown in Fig. 12 by solid lines. Figure 15 shows a CMP gather with the predicted traveltimes of the sea-bottom and peg-leg multiples indicated by solid lines.

Conclusions

A new method for predicting kinematic properties of different kinds of multiples and peg-leg reflections in unstacked seismic data is proposed. The method is based on the homeomorphic-imaging (HI) technique. The prediction is made using a two-step process. In the first step, the values for the angle of emergence and radii of curvature of the wavefront for primary reflections from multiple-generating interfaces are obtained;

![Figure 12](image_url). A stacked section of real marine data. The predicted zero-offset times of the first surface multiple and a strong peg-leg from the reflector M are indicated by solid lines.
Figure 13. (a) Estimated angle of emergence of the wavefront for reflection from the sea-bottom with and without smoothing. (b) Estimated radius of curvature of the wavefront for reflection from the sea-bottom with and without smoothing.
Figure 14. (a) Estimated angle of emergence of the wavefront from reflector M with and without smoothing. (b) Estimated radius of curvature of the wavefront from reflector M with and without smoothing.

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these parameters are estimated for every source point directly from unstacked data using the HI technique. The second step consists of multiple prediction from primary reflectors that satisfy the so-called multiple condition; this condition is the equality of the absolute values of the angles of emergence calculated from the first step.

The main advantages of the proposed method are:

1. No information on subsurface geology is required.
2. The proposed method is equally valid for all types of multiples (water-bottom, peg-leg and interbed), provided it is possible to pick primaries from multiple-generating interfaces on a CMP stack section.

The synthetic and numerical examples given demonstrate the power of the proposed method.

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